Announcements

- Quiz 3 will be on feb 11 thursday on sections 3.1 and 3.2
- No calculators for this quiz. You have to find the determinants using the methods we have learned.
- Please check the homework problems posted and make sure you have the updated list.

From last class

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$

From last class

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$

then write the following matrix.

$$\operatorname{adj} A = \begin{bmatrix} C_{11} & C_{21} & C_{23} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix},$$

Note carefully how the subscripts appear for the entries of A and the entries of the new matrix adj A. They are flipped.

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5. Repeat the same for the remaining 2 rows. (The cofactors with alternating aigns form the next 2 columns)
6. Divide adj A by det A and this gives A^{-1}.

Example 16, Sec 3.3

Compute the adjugate and use it to find A^{-1}.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & -3 & 1 \\ 0 & 0 & 3 \end{bmatrix},$$

Write the 9 cofactors with sign alternating

Example 16, Sec 3.3

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$$C_{11} = +\underbrace{\begin{vmatrix} -3 & 1 \\ 0 & 3 \end{vmatrix}}_{-9}$$

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Example 14, Sec 3.3

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$$\begin{array}{ccc} 1 & 2 & 4 \\ 0 & -3 & 1 \\ 0 & 0 & 3 \end{array}$$

$$C_{31} = + \underbrace{\begin{vmatrix} 2 & 4 \\ -3 & 1 \end{vmatrix}}_{14}$$

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Now you have all cofactors with proper signs. Write them down as columns. Be careful about terms with double negatives.

$$\text{adj}\, A = \begin{bmatrix} -9 & -6 & 14 \\ 0 & 3 & -1 \\ 0 & 0 & -3 \end{bmatrix},$$

To find A^{-1}, we have to find $\det A$. Here $\det A = -9$ (Why?). So

$$A^{-1} = (-\frac{1}{9}) \begin{bmatrix} -9 & -6 & 14 \\ 0 & 3 & -1 \\ 0 & 0 & -3 \end{bmatrix},$$

Determinants and Geometry

Theorem

If A is a 2×2 matrix, the area of the parallelogram determined by the columns of A is $|\det A|$ (the absolute value of $\det A$, since area is never negative).

Steps to find area if 4 vertices ae given

1. We have to first translate the given parallelogram to one with the origin as a vertex.
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3. Now the parallelogram is identified by the columns of a 2×2 matrix whose columns are any two "new" vertices written as columns.
4. Find the determinant of this matrix, drop the negative sign.

Example 20, sec 3.3

Find the area of the parallelogram whose vertices are (0,0), (-1,3), (4,-5) and (3,-2).

Solution: Here one vertex is already (0,0). So we can write the 2×2 matrix as (You can choose any 2 vertices other than the origin)

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<u>Solution:</u> Here one vertex is already (0,0). So we can write the 2×2 matrix as (You can choose any 2 vertices other than the origin)

$$A = \begin{bmatrix} -1 & 4 \\ 3 & -5 \end{bmatrix},$$

So $\det A = -7$ and the area of the parallelogram is 7 sq units.

Example 22, sec 3.3

Find the area of the parallelogram whose vertices are (0,-2), (6,-1), (-3,1) and (3,2).

Solution: Here we have to translate the parallelogram so that one vertex is (0,0).

Example 22, sec 3.3

Find the area of the parallelogram whose vertices are (0,-2), (6,-1), (-3,1) and (3,2).

Solution: Here we have to translate the parallelogram so that one vertex is (0,0).
Choose (0,-2) (or any one you like) and subtract all four vertices from this.

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We get (0,0), (6,1), (-3,3) and (3,4)

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Choose (0,-2) (or any one you like) and subtract all four vertices from this.
We get (0,0), (6,1), (-3,3) and (3,4)
So we can write the 2×2 matrix as (You can choose any 2 vertices, again don't choose origin)

$$A = \begin{bmatrix} 6 & -3 \\ 1 & 3 \end{bmatrix}.$$

So $\det A = 21$ and the area of the parallelogram is 21 sq units.

Area of any triangle with vertices given

Let R be a triangle with vertices at (x_1, y_1), (x_2, y_2) and (x_3, y_3). Then the area of the triangle is given by

$$\frac{1}{2}\left|\det\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}\right|.$$

Here the vertical lines denote the absolute value of the determinant of the matrix shown.

Example

Find the area of a triangle whose vertices are (1,2), (3,4) and (2,8)

Solution:

$$\frac{1}{2}\left|\det\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 1 \\ 2 & 8 & 1 \end{bmatrix}\right| = \frac{1}{2}((1)(-4) - (2)(1) + (1)(16)) = \frac{1}{2}(10) = 5.$$

Question

Suppose you are given 3 points and asked to find the area of a triangle with those points as vertices. What is your conclusion if the determinant in the area formula is 0?

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Suppose you are given 3 points and asked to find the area of a triangle with those points as vertices. What is your conclusion if the determinant in the area formula is 0?

Answer: The 3 points cannot form a triangle. They all lie on a straight line or the 3 points are collinear.

Example

Consider (1,1), (2,2) and (3,3) and see what happens!

Points in $\mathbb{R}^3$ to be coplanar

Let (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) be any 3 points in $\mathbb{R}^3$. If

$$\det \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} = 0,$$

we say that these 3 points are in the same plane (coplanar).

Equation of a straight line

Let (x_1, y_1), and (x_2, y_2) be any 2 distinct points in $\mathbb{R}^2$. Then

$$\det \begin{bmatrix} 1 & x & y \\ 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \end{bmatrix} = 0$$

gives the equation of the straight line passing through these points.

Example

Find the equation of the straight line passing through the points (1,3), and (2,-4).

Solution: Then

$$\det \begin{bmatrix} 1 & x & y \\ 1 & 1 & 3 \\ 1 & 2 & -4 \end{bmatrix} = 0$$

$$(1)(-10) - x(-7) + y(1) = 0$$

$$7x + y - 10 = 0$$

Equation of a Plane

Let (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) be any 3 points in $\mathbb{R}^3$. The equation of a plane passing through these points is

$$\det \begin{bmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{bmatrix} = 0,$$

This gives an equation of the form $Ax + By + Cz = D$ where A, B, C and D are numbers depending on the given vertices. (You learn a more elegant way to do this in calculus, using normal vectors)

How to use TI-89 calculator in Linear Algebra?

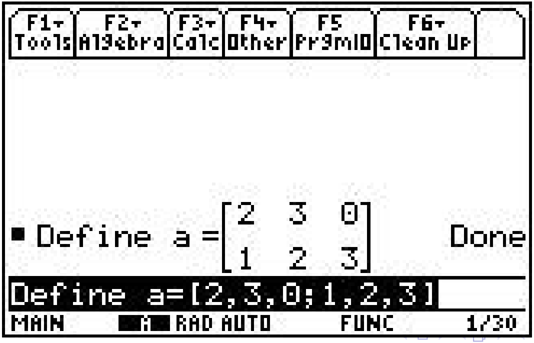
1. TI-89 calculators have useful linear algebra options that will help you with matrix/determinant calculations.
2. Very useful in classes whose objective is to calculate and move on (esp in engineering, economics, chemistry etc)
3. For a MA2321 exam/quiz you could use it to check your answer but if "No Calculator" is specified, you must show your work for that problem.
4. If you use it on a test/quiz please be familiar with the options beforehand. Trying to master the "User's Manual" during the test/quiz will not work out.

TI89- The "Home Screen" way

How to enter a matrix from home screen?

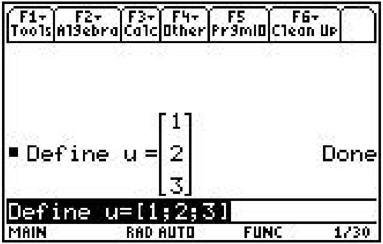
Use the **Define** option. You could do this by pressing F4 and option 1 or typing it in. Suppose you want $a = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \end{bmatrix}$,

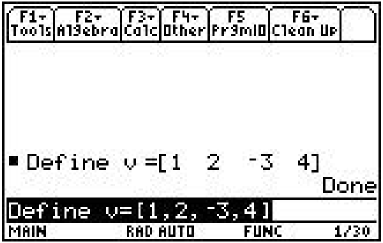
1. Separate columns by commas
2. Separate rows by semi-colons
3. Put [and] before and after.



TI89- More Examples

Similarly you enter $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $v = \begin{bmatrix} 1 & 2 & -3 & 4 \end{bmatrix}$ as follows.

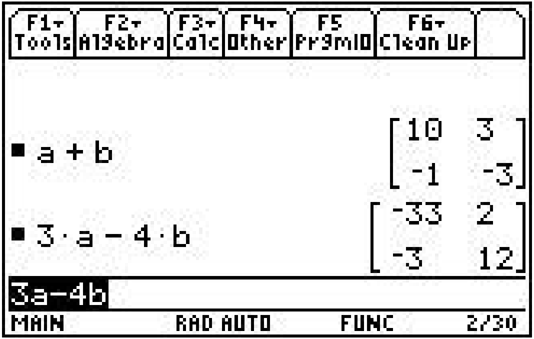


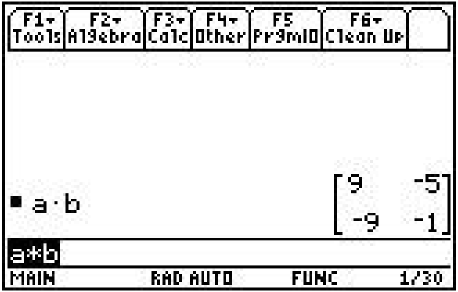


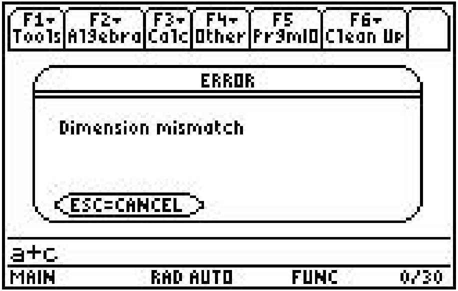
TI89- Matrix Algebra

Suppose $a = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$, $b = \begin{bmatrix} 9 & 1 \\ 0 & -3 \end{bmatrix}$ and $c = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & 1 \\ -2 & 0 & 3 \end{bmatrix}$.

Enter these matrices the same way. Suppose we want to find $a + b$, $3a - 4b$, ab and $a + c$



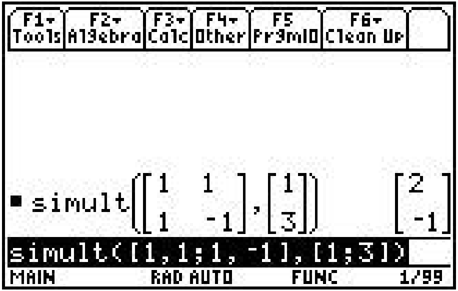

F1 Tools
F2 Algebra
F3 Calc
F4 Other
F5 PrgmIO
F6 Clean Up
■ a·b
$\begin{bmatrix} 9 & -5 \\ -9 & -1 \end{bmatrix}$
a*b
MAIN RAD AUTO FUNC 1/30


F1 Tools
F2 Algebra
F3 Calc
F4 Other
F5 PrgmIO
F6 Clean Up
ERROR
Dimension mismatch
ESC=CANCEL
a+c
MAIN RAD AUTO FUNC 0/30

TI89- Solving simultaneous equations

Suppose you want to solve the following on your TI-89. Press **MATH** and then Option 4 and then Option 5. This starts a line simult(. Here you fill your coefficient matrix and the vector of right hand side one after the other, close)

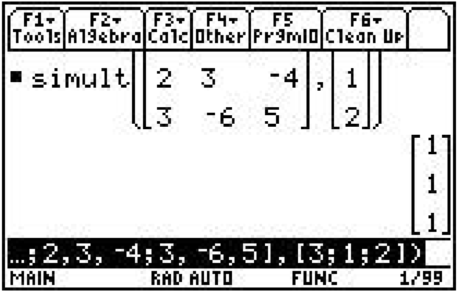
$$\begin{cases} x + y = 1 \\ x - y = 3 \end{cases}$$



TI89- Solving simultaneous equations

Three equations, 3 variables. Screen re-adjusts.

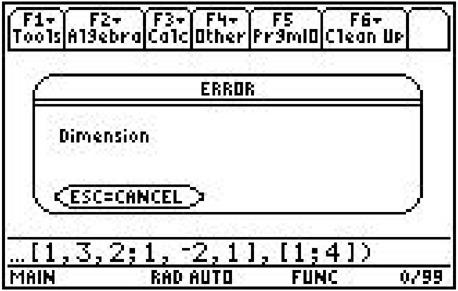
$$\begin{cases} x + y + z = 3 \\ 2x + 3y - 4z = 1 \\ 3x - 6y + 5z = 2 \end{cases}$$



TI89- Solving simultaneous equations

What if there are free variables? TI-89 freaks out!!

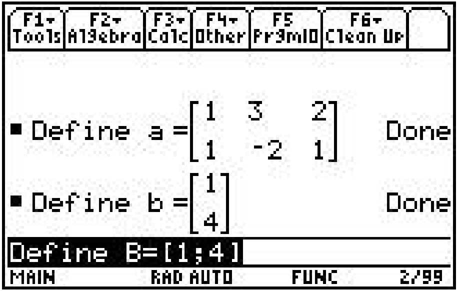
$$\begin{cases} x + 3y + 2z = 1 \\ x - 2y + z = 4 \end{cases}$$



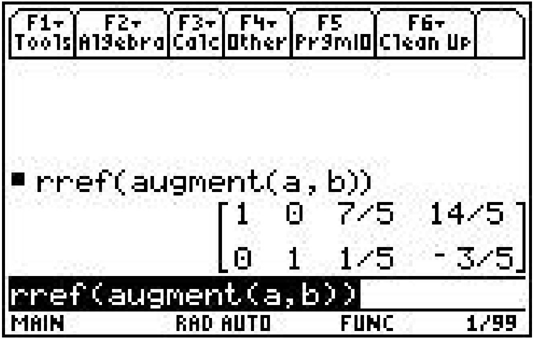
TI89- Solving simultaneous equations

Want to do elementary row operations. Write A and B as usual and then apply **rref(augment(A,B))**. (You could press **MATH** and Option 4 and then Option 4 for RREF and **MATH** and Option 4 and then Option 7 for augment).

$$\begin{cases} x + 3y + 2z = 1 \\ x - 2y + z = 4 \end{cases}$$



TI89- Solving simultaneous equations



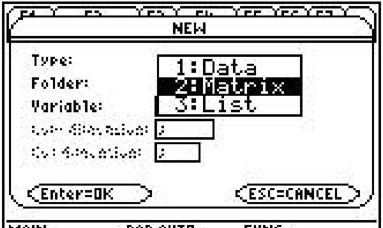
TI-89 WILL NOT interpret this any further. YOU must decide free variables, basic variables etc and do the rest. Keep in mind that the last column is the augmented column.

Other useful stuff

1. To find transpose of a matrix, use **MATH** and then Option 4 and then Option 1 for transpose
2. If you want to find A^{-1}, you type $A \wedge (-1)$. (Will return **error: singular matrix** if not invertible).
3. To get the 3×3 identity matrix, use **MATH** and then Option 4 and then Option 6. This displays **identity(**, then type 3 and then evaluate. (You could just type in the 3×3 identity matrix as well)
4. **det(a)** computes the determinant of the matrix A.

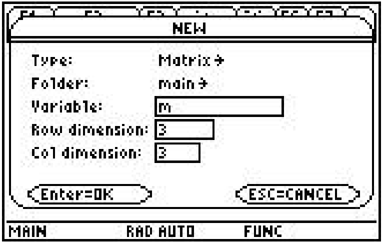
TI89- "Sophisticated" matrix entry

If you want to explore all features of your TI-89, you could choose **APPS** option 6. Select "New", on the next screen choose "Matrix" for type, enter a name for the matrix and its desired size. Here are the screenshots.

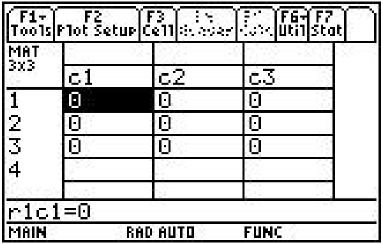


TI89- "Sophisticated" matrix entry

I choose "m" for the name.

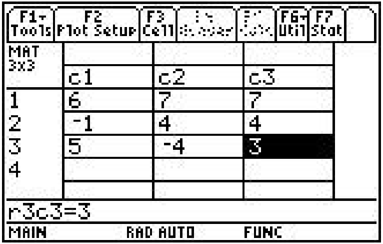


This is what you see in the beginning



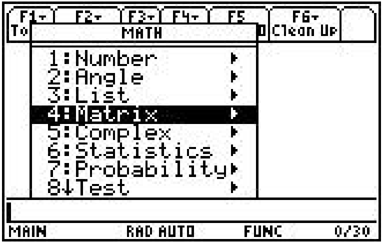
TI89- "Sophisticated" matrix entry

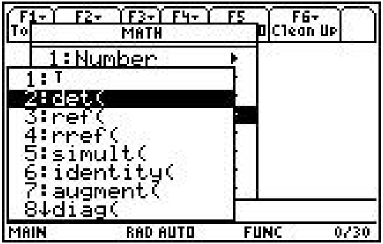
Fill each slot with your numbers.



If you want to enter negative numbers use the "-" key to the left of the "enter" key (NOT the subtraction key)

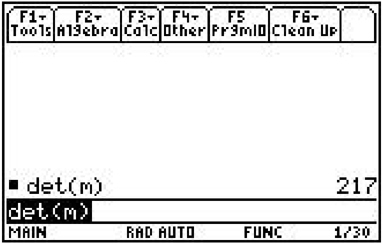
TI89- The "MATH- option 4" choices

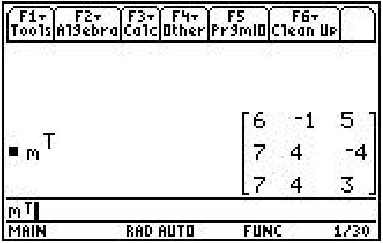




Results of det, transpose

Some results





Results of inverse, RREF

Some results